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The coupling constants $g_{\rho\pi\gamma}$ and $g_{\omega\pi\gamma}$ as derived from QCD sum rules

A. Gökalp[a], O. Yılmaz[b]

Physics Department, Middle East Technical University, 06531 Ankara, Turkey

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Abstract. We employ QCD sum rules to calculate the coupling constants $g_{\rho\pi\gamma}$ and $g_{\omega\pi\gamma}$ by studying the three point $\rho\pi\gamma$ and $\omega\pi\gamma$ correlation functions. Our results are in good agreement with the coupling constants $g_{\rho\pi\gamma}$ and $g_{\omega\pi\gamma}$ that are deduced utilizing the experimentally measured values of the $\Gamma(\rho^0 \to \pi^0\gamma)$ and $\Gamma(\omega \to \pi^0\gamma)$ decay widths.

Radiative transitions of the type $V \to P\gamma$ where V and P belong the lowest multiplets of vector (V) and pseudoscalar (P) mesons have been the subject of continuous interest both theoretically and experimentally [1]. These transitions have been considered from the points of view of a large variety of theoretical models, such as phenomenological quark models [2], potential models [3], bag models [4], and effective Lagrangian approaches [5]. All these approaches provide effective methods of investigation of these hadronic phenomena for which a formulation for the application of QCD from the first principles has not been possible so far. The effective Lagrangian approach provide a framework to study in general the physics of light neutral vector mesons, ρ^0, ω and ϕ, by combining the vector meson dominance and chiral dynamics which are the two principles governing low energy QCD in a suitably constructed effective Lagrangian [6].

On the other hand, the vector meson–pseudoscalar meson–photon $VP\gamma$ vertex also plays a role in photoproduction reactions of vector mesons on nucleons. Although at sufficiently high energies and low momentum transfers electromagnetic production of vector mesons on nucleon targets has been explained by pomeron exchange models, at low energies near threshold scalar and pseudoscalar meson exchange mechanisms become important [7]. For the photoproduction reactions involving ρ^0 and ω mesons, the effective coupling constants $g_{\rho\pi\gamma}$ and $g_{\omega\pi\gamma}$ are among the physical inputs that are used in the analyses of these reactions. In these studies, an effective Lagrangian describing the $VP\gamma$ vertex is assumed, which also defines the coupling constant $g_{VP\gamma}$, and these coupling constants are then determined utilizing the experimental decay widths $\Gamma(V \to P\gamma)$ of the vector mesons. However, it should be noted that in these decays the four-momentum of the pseudoscalar meson P is time-like, ${p'}^2 > 0$, whereas in the pseudoscalar exchange amplitude contributing to the photoproduction of vector mesons it is space-like, ${p'}^2 < 0$. Thus, if the coupling constants determined from the decays $V \to P\gamma$ are used in scalar and pseudoscalar exchange amplitudes, a long extrapolation in momentum transfer is assumed, which may be questionable. Therefore, it is of interest to study the effective coupling constants $g_{VP\gamma}$ from another point of view as well.

In this work, we estimate the coupling constants $g_{\rho\pi\gamma}$ and $g_{\omega\pi\gamma}$ by employing QCD sum rules which provide an efficient and model-independent method to study many hadronic observables, such as decay constants and form factors, in terms of nonperturbative contributions proportional to the quark and gluon condensates [8,9]. Using the techniques of QCD sum rules, the nonperturbative QCD physics is incorporated systematically as power corrections in the short-distance operator product expansion.

In order to derive the QCD sum rule for the coupling constant $g_{V\pi\gamma}$ where V denotes ρ^0 or ω meson, we begin by considering the three point correlation function

$$T_{\mu\nu}(p,p') = \int \mathrm{d}^4x\mathrm{d}^4y \mathrm{e}^{\mathrm{i}p'\cdot y}\mathrm{e}^{-\mathrm{i}p\cdot x}\langle 0|T\{j^\gamma_\mu(0) j^V_\nu(x) j_5(y)\}|0\rangle, \quad (1)$$

where the interpolating currents j^V_ν forρ^0 and ω meson are $j^\rho_\nu = (1/2)(\overline{u}\gamma_\nu u - \overline{d}\gamma_\nu d)$, $j^\omega_\nu = (1/6)(\overline{u}\gamma_\nu u + \overline{d}\gamma_\nu d)$, respectively, $j_5 = (1/2)(\overline{u}\mathrm{i}\gamma_5 u - \overline{d}\mathrm{i}\gamma_5 d)$ is the interpolating current for π^0 [8], and $j^\gamma_\mu = e_u\overline{u}\gamma_\mu u + e_d\overline{d}\gamma_\mu d$, where e_u and e_d denote the quark charges, is the quark electromagnetic current. In accordance with QCD sum rule techniques, we consider the three point correlation function $T_{\mu\nu}(p,p')$ in the Euclidean region defined by $p^2 = -Q^2 \sim -1\,\mathrm{GeV}^2$, ${p'}^2 = -{Q'}^2 \sim -1\,\mathrm{GeV}^2$.

The theoretical part of the sum rule for the coupling constant $g_{V\pi\gamma}$ is obtained in terms of QCD degrees of freedom by calculating the perturbative contribution and the power corrections from operators of different dimen-

[a] e-mail: agokalp@metu.edu.tr
[b] e-mail: oyilmaz@metu.edu.tr

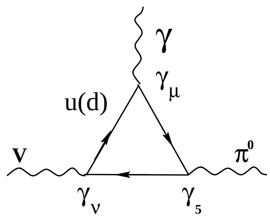

Fig. 1. Bare quark loop diagram for $V\pi\gamma$ vertex

sions to the three point correlation function. In the region $Q^2, Q'^2 \sim 1\,\mathrm{GeV}^2$ the perturbative contribution can be approximated by the lowest order free-quark loop diagram shown in Fig. 1. Furthermore, we consider the power corrections from operators of different dimensions, resulting in contributions to the three point correlation function that are proportional to the terms $\langle\overline{\psi}\psi\rangle$, $\langle\overline{\psi}\sigma\cdot G\psi\rangle$ and $\langle(\overline{\psi}\psi)^2\rangle$. We do not consider the gluon condensate contribution proportional to $\langle G^2\rangle$ since it is estimated to be negligible for light-quark systems. The calculations of the power corrections are performed in the fixed point gauge [10]. We work in the SU(2) flavor context with $m_u = m_d = m_q$; moreover, we perform our calculations of the perturbative and power correction contributions in the limit $m_q = 0$. In this limit, the perturbative bare-loop diagram does not make any contribution, and only operators of dimensions $d = 3$ and $d = 5$ make contributions that are proportional to $\langle\overline{\psi}\psi\rangle$ and $\langle\overline{\psi}\sigma\cdot G\psi\rangle$, respectively. The relevant Feynman diagrams for the calculation of these power corrections are shown in Figs. 2 and 3.

The contribution coming from the operator of dimension $d = 3$ can be calculated from Fig. 2a to be

$$D_{3a} = \frac{3}{4} e_q g_V \langle\overline{\psi}_\alpha(0)\psi_\beta(x)\rangle \left(\gamma_5 \frac{1}{\not p'}\gamma_\mu \frac{1}{\not p}\gamma_\nu\right)_{\alpha\beta}, \quad (2)$$

where $g_V = 1$ for the ρ meson and $g_V = 1/3$ for the ω meson, resulting from the choice of the interpolating currents, and ψ is the quark field. Since the quark condensate can be expressed by expanding the quark field $\psi(x)$ to first order in the form [11]

$$\langle\overline{\psi}_\alpha(0)^a \psi_\beta(x)^b\rangle = \frac{1}{12} g_{\alpha\beta} g^{ab}\langle\overline{\psi}\psi\rangle + \frac{\mathrm{i}m_q}{48} g^{ab}\langle\overline{\psi}\widehat{x}_{\beta\alpha}\psi\rangle, \quad (3)$$

with a, b denoting color indices, we obtain the contribution D_{3a} as

$$D_{3a} = \mathrm{i}\frac{3}{4} e_q g_V \frac{1}{p'^2}\frac{1}{p^2}\varepsilon_{\alpha\beta\mu\nu} p_\alpha p'_\beta \langle\overline{\psi}\psi\rangle. \quad (4)$$

The quark condensate contributions coming from the diagrams in Fig. 2b,c can be calculated similarly, however, they do not give any contribution after double Borel transform.

The contributions of the operators of dimension $d = 5$ are calculated using the diagrams shown in Fig. 3. The contribution resulting from the diagram in Fig. 3b with one gluon line emitted can be written as

$$D_{5b} = e_q g_V \left\langle \overline{\psi}^i_\sigma(0) G^a_{\rho\alpha} \psi^j_\delta(0) \left(\frac{\lambda^a}{2}\right)^{ij} \right\rangle$$
$$\times \frac{g_s}{8}\left(\gamma_5 \frac{1}{\not p'}\gamma_\mu \frac{\mathrm{d}}{\mathrm{d}k_\rho}\frac{1}{\not p + \not k}\gamma_\alpha \frac{1}{\not p}\gamma_\nu\right)_{\sigma\delta}, \quad (5)$$

and by utilizing the relation [11]

$$\langle\overline{\psi}^i_\sigma G^a_{\rho\alpha}\psi^j_\delta\rangle = C(\sigma_{\rho\alpha})_{\delta\sigma}\left(\frac{\lambda^a}{2}\right)^{ji} \quad (6)$$

with the coefficient

$$C = \frac{1}{384}\left\langle \overline{\psi}\sigma_{\rho\alpha} G^a_{\rho\alpha}\frac{\lambda^a}{2}\psi\right\rangle \equiv \frac{1}{384}\langle\overline{\psi}\sigma\cdot G\psi\rangle \quad (7)$$

we obtain this contribution in the form

$$D_{5b} = -\frac{\mathrm{i}}{32} e_q g_V \frac{1}{p'^2}\frac{1}{p^4}\varepsilon_{\alpha\beta\mu\nu} p_\alpha p'_\beta \langle\overline{\psi}\sigma\cdot G\psi\rangle. \quad (8)$$

The contribution coming from the diagram in Fig. 3c can be calculated in a similar way to be

$$D_{5c} = -\frac{3\mathrm{i}}{32} e_q g_V \frac{1}{p^2}\frac{1}{p'^4}\varepsilon_{\alpha\beta\mu\nu} p_\alpha p'_\beta \langle\overline{\psi}\sigma\cdot G\psi\rangle. \quad (9)$$

However, it should be noted that in fixed point gauge, the momentum is not conserved in a chosen fixed point. After calculations the momentum of the soft external fields are set equal to zero and thus a gauge invariant result is obtained, but the same point as the fixed point should be chosen for all diagrams in calculations [10]. In the case of the diagram in Fig. 3a the contribution proportional to $\langle\overline{\psi}\sigma\cdot G\psi\rangle$ results from the expansion of $\overline{\psi}(0)\psi(x)$ to second order and the terms $\langle x_\alpha x_\beta \nabla_\alpha \nabla_\beta \overline{\psi}(0)\psi(0)\rangle$ can be rewritten in terms of $\langle\overline{\psi}\sigma\cdot G\psi\rangle$ [11], and in this way for the contribution of this diagram we obtain

$$\begin{aligned} D_{5a} &= \frac{3}{256} e_q g_V g_s \langle\overline{\psi}\sigma\cdot G\psi\rangle \left(-\mathrm{i}\frac{\partial}{\partial p_\lambda}\right)\left(-\mathrm{i}\frac{\partial}{\partial p_\delta}\right) \\ &\quad \times \mathrm{Tr}\left(g_{\lambda\delta}\gamma_5 \frac{1}{\not p'}\gamma_\mu \frac{1}{\not p}\gamma_\nu\right) \\ &= \frac{3\mathrm{i}}{16} e_q g_V g_s \frac{1}{p'^2}\frac{1}{p^4}\varepsilon_{\alpha\beta\mu\nu} p_\alpha p'_\beta \langle\overline{\psi}\sigma\cdot G\psi\rangle. \end{aligned} \quad (10)$$

We then turn to the calculation of the three point correlation function through phenomenological considerations. The vertex function $T_{\mu\nu}(p, p')$ satisfies a double dispersion relation. In general such a dispersion relation can be written in three ways by choosing two of the three channels. For our purpose, we choose the vector and pseudoscalar channels and by saturating this dispersion relation by the lowest lying meson states in these channels we obtain the physical part of the sum rule as

$$T_{\mu\nu}(p, p') = \frac{\langle 0|j^V_\nu|V\rangle\langle V(p)|j^\gamma_\mu|\pi(p')\rangle\langle\pi|j_5|0\rangle}{(p^2 - m_V^2)(p'^2 - m_\pi^2)} + \cdots, \quad (11)$$

where the contributions from the higher states and the continuum is denoted by dots. In this expression, the overlap amplitudes for vector and pseudoscalar mesons are $\langle 0|j^V_\nu|V\rangle = \lambda_V u_V$ where u_V is the polarization vector of

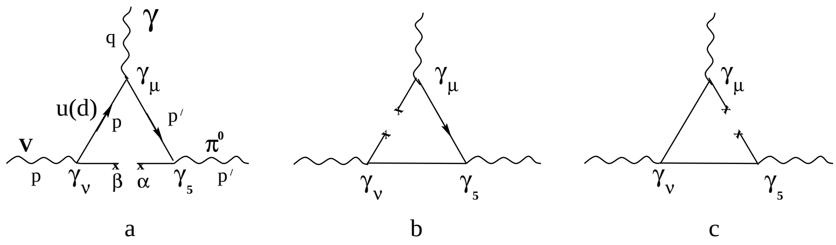

Fig. 2a–c. Operators of dimension 3 corrections proportional to $\langle\overline{\psi}\psi\rangle$

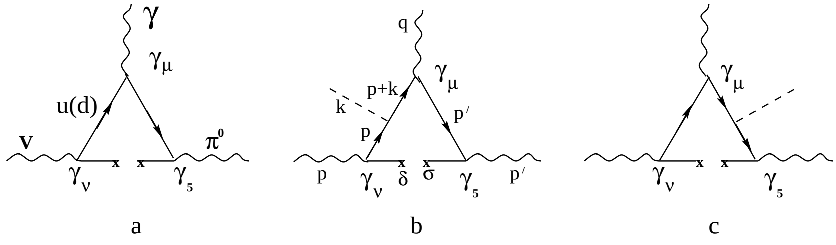

Fig. 3a–c. Operators of dimension 5 corrections proportional to $\langle\overline{\psi}\sigma\cdot G\psi\rangle$. The dotted lines denote gluons

the vector meson and $\langle\pi|j_5|0\rangle = \lambda_\pi$. The matrix element of the electromagnetic current is given by

$$\langle V(p)|j_\mu^\gamma|\pi(p')\rangle = -\mathrm{i}\frac{e}{m_V} g_{V\pi\gamma} K(q^2)\varepsilon^{\mu\alpha\beta\delta} p_\alpha u_\beta q_\delta, \quad (12)$$

where $q = p - p'$ and $K(0) = 1$. This expression defines the coupling constant $g_{V\pi\gamma}$ through the effective Lagrangian

$$\mathcal{L}^{\text{eff.}}_{V\pi\gamma} = \frac{e}{m_V} g_{V\pi\gamma}\varepsilon^{\mu\nu\alpha\beta}\partial_\mu V_\nu \partial_\alpha A_\beta \pi^0 \quad (13)$$

describing the $V\pi\gamma$ vertex [7].

After performing the double Borel transform with respect to the variables Q^2 and Q'^2, we obtain the sum rule for the coupling constant $g_{V\pi\gamma}$

$$g_{V\pi\gamma} = \frac{m_V}{\lambda_V\lambda_\pi}\mathrm{e}^{m_V^2/M^2}\mathrm{e}^{m_\pi^2/M'^2}\frac{g_V}{e}\left(e_u\langle\overline{u}u\rangle \pm e_d\langle\overline{d}d\rangle\right)$$
$$\times\left(-\frac{3}{4} + \frac{5}{32}m_0^2\frac{1}{M^2} - \frac{3}{32}m_0^2\frac{1}{M'^2}\right), \quad (14)$$

where we use the relation $\langle\overline{\psi}\sigma\cdot G\psi\rangle = m_0^2\langle\overline{\psi}\psi\rangle$. The plus sign is for the ρ meson and the minus sign is for the ω meson. For the numerical evaluation of the sum rule we use the values $m_0^2 = (0.8\pm 0.02)\,\mathrm{GeV}^2$, $\langle\overline{u}u\rangle = \langle\overline{d}d\rangle = (-0.014\pm 0.002)\,\mathrm{GeV}^3$ [12], and $m_\rho = 0.770\,\mathrm{GeV}$, $m_\omega = 0.782\,\mathrm{GeV}$, $m_{\pi^0} = 0.135\,\mathrm{GeV}$ [13]. For the overlap amplitude for the vector meson states, we use the values that are obtained from the experimental leptonic decay widths [13] by noting that neglecting the electron mass the e^+e^- decay width of the vector meson is given by

$$\Gamma(V\to e^+e^-) = \frac{4\pi\alpha^2}{3}\frac{\lambda_V^2}{m_V^3},$$

and in this way we obtain the values $\lambda_\rho = (0.118\pm 0.003)\,\mathrm{GeV}^2$ and $\lambda_\omega = (0.036\pm 0.001)\,\mathrm{GeV}^2$. We note that these values do obey the SU(3) relation $\lambda_\rho = 3\lambda_\omega$ within 10% accuracy. The overlap amplitude λ_π for the π meson state is given by the relation $\lambda_\pi = f_\pi m_\pi^2/(m_u + m_d)$ [14]. We use the experimental value $f_\pi = 0.132\,\mathrm{GeV}$ and the physical mass $m_{\pi^0} = 0.135\,\mathrm{GeV}$ along with $m_u + m_d = (0.0128\pm 0.0025)\,\mathrm{GeV}$ [15], and obtain this amplitude as $\lambda_\pi = (0.196\pm 0.038)\,\mathrm{GeV}^2$. In order to analyze the dependence of the coupling constant $g_{V\pi\gamma}$ on the Borel parameters M^2 and M'^2, we study independent variations of M^2 and M'^2 in the interval $0.6\,\mathrm{GeV}^2 \le M^2, M'^2 \le 1.4\,\mathrm{GeV}^2$ as these limits determine the allowed interval for the vector channel [16]. The variation of the coupling constant $g_{\rho\pi\gamma}$ and $g_{\omega\pi\gamma}$ as a function of the Borel parameters M^2 for different values of M'^2 is shown in Figs. 4 and 5, respectively. The examination of these figures indicate that the sum rule is quite stable with these reasonable variations of M^2 and M'^2. Besides those due to variations of M^2 and M'^2, the other sources contributing to the uncertainty in the coupling constants are the uncertainties in the estimated values of the vacuum condensates. If we take these uncertainties into account by a conservative estimate, we obtain the coupling constants $g_{\rho\pi\gamma} = 0.63\pm 0.14$ and $g_{\omega\pi\gamma} = 1.85\pm 0.38$. These values of the coupling constant are consistent with their values used in the analysis of ρ^0 and ω photoproduction reactions through pseudoscalar exchange amplitudes which are $g_{\rho\pi\gamma} = 0.54$ and

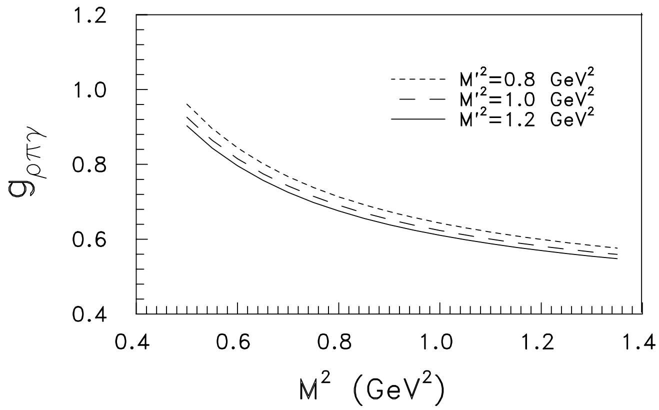

Fig. 4. The coupling constant $g_{\rho\pi\gamma}$ as a function of the Borel parameter M^2 for different values of M'^2

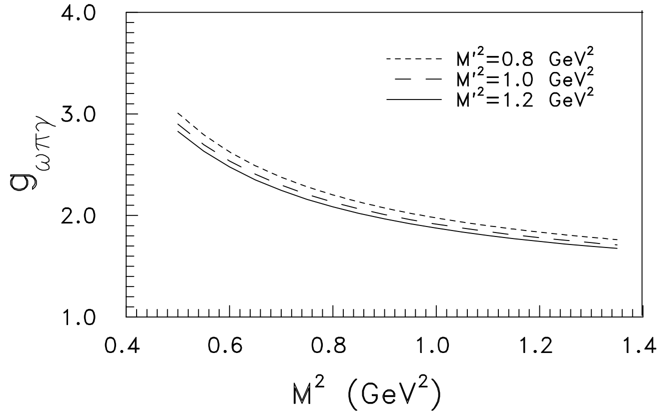

Fig. 5. The coupling constant $g_{\omega\pi\gamma}$ as a function of the Borel parameter M^2 for different values of M'^2

$g_{\omega\pi\gamma} = 1.82$, respectively [17]. Moreover, if we use the effective Lagrangian given in (4), then the decay width for $V \to \pi^0\gamma$ is obtained as

$$\Gamma(V \to \pi^0\gamma) = \frac{\alpha}{24}\frac{(m_V^2 - m_{\pi^0}^2)^3}{m_V^5} g_{V\pi\gamma}^2 . \tag{15}$$

The measured decay widths $\Gamma(\rho^0 \to \pi^0\gamma) = (102 \pm 26)$ keV and $\Gamma(\omega \to \pi^0\gamma) = (717 \pm 49)$ keV [13], which roughly follow the SU(3) prediction as regards their ratio, can then be utilized to obtain the coupling constants $g_{\rho\pi\gamma}$ and $g_{\omega\pi\gamma}$ as $g_{\rho\pi\gamma} = 0.69 \pm 0.09$ and $g_{\omega\pi\gamma} = 1.82 \pm 0.06$. Our results, therefore, are in good agreement with the coupling constants deduced from the experimental values of these decay widths. We also note that the electromagnetic decays $V \to P\gamma$ of vector mesons in the flavor SU(3) sector was studied previously [14] by employing the method of QCD sum rules in the presence of the external electromagnetic field. Our results, which are obtained by QCD sum rules utilizing three point correlation functions, are consistent with the values obtained in that analysis and therefore supplement the study of these decays using the QCD sum rule method.

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